



Recreational Mathematics Colloquium I

BOOK OF ABSTRACTS

University of Évora
April 29th - May 2nd, 2009



Organization:

Ludus Association
University of Évora
Ciência Viva
Center of History of Science of the University of Lisbon
Portuguese Mathematical Society
Museum of Science, University of Lisbon

Organising Committee:

Alda Carvalho (ISEL, Portugal)
Carlos Santos (ISEC, Portugal)
Jorge Nuno Silva (University of Lisbon, Portugal)
Paulo Infante (University of Évora, Portugal)
Sandra Vinagre (University of Évora, Portugal)

Scientific Committee:

Aviezri Fraenkel (Weizmann Institute of Science, Israel)
David Wolfe (Gustavus Adolphus College, Canada)
João Pedro Neto (University of Lisbon, Portugal)
Jorge Buescu (University of Lisbon, Portugal)
Jorge Nuno Silva (University of Lisbon, Portugal)
Michele Emmer (University of Rome, Italy)
Nuno Crato (Technical University of Lisbon, Portugal)
Richard Guy (University of Calgary, Canada)
Richard Nowakowski (Dalhousie University, Canada)
Thomas Banchoff (Brown University, USA)

Sponsors:



Contents

1	Foreword	3
2	Invited Speakers	5
3	Program	11
4	Abstracts	15
5	Mailing List	28

1 Foreword

“Recreational Mathematics” is a problematic expression. For some people, like most professional mathematicians, Mathematics is lots of fun; but for others, like some students, Mathematics can be a nightmare.

Historically, we know that some mathematical research areas are deeply linked to puzzles and games, probability and chance games, graph theory and the Brigdes of Königsberg come to mind.

The University of Évora, the Ludus Association, the Museum of Science of the University of Lisbon, the Portuguese Mathematical Society and the Center of History of Science of the University of Lisbon will organize the Recreational Mathematics Colloquium I.

Our Colloquium will be a Show and Tell of bright pearls of Mathematics, with varied levels of sophistication, entertaining many audiences. Its main goal is to foster mathematical appreciation, an important step if we are to see improvements in its practice. More information can be consulted in the following web page:

<http://ludicum.org/rm09/>

The Organising Committee

2 Invited Speakers

AVIEZRI FRAENKEL (Weizmann Institute of Science, Israel)

Aviezri Fraenkel is a professor of Computer Science and Applied Mathematics at the Weizmann Institute of Science, Rehovot, Israel, currently Gorenstein Visiting Professor at Queens College, Queens, NY. He has published 200 papers and book chapters in mathematics, computer science, information storage and retrieval, and Judaica.

He is a member of the Editorial Board of Discrete Math., Electronic J. of Combinatorics, Theoretical Computer Science, Internat. J. of Appl. Math., Internat. Computer Games Assoc. J.; and on the Advisory Board of INTEGERS Electronic J. of Combinatorial Number Theory. Member of the Electronic Publishing Committee of the Europ. Math. Soc.

Selected Honors: Feder Foundation Prize for initiating and creating the Responsa Project (1972); Quality Initiative Citation to the Responsa Project for creative, high-quality and visionary projects to celebrate the 50th anniversary of the establishment of the State of Israel (1998); The Fraenkel Festschrift: Electronic J. Combinatorics, vol. 8(2); Euler Medal recipient of the Institute for Combinatorics and Its Applications 2005; Recipient of WEIZAC Medal for the WEIZAC computer project, designated as IEEE Milestone (2006); Israel Prize to the Responsa Project (2007).

DAVID WOLFE (Gustavus Adolphus College, Canada)

Experience:

Gustavus Adolphus College, Saint Peter, MN, Associate Professor: 2001 to present, Department of Mathematics and Computer Science.

University of California, Berkeley, Lecturer: Spring 1992 to Spring 1996, Division of Computer Science.

Post-doc: Fall 1993 to Spring 1994, Department of Mathematics.

Education:

University of California, Berkeley: Ph.D. in Computer Science, 1991.

Cornell University: B.S. in Electrical Engineering, 1985.

Interests:

Discrete mathematics and computer science, Game theory, Probability and stochastic processes, Recreational mathematics, Combinatorics, Queueing theory, Randomness and Computation.

Publications:

Lessons in Play: An Introduction to the Combinatorial Game Theory, with Michael Albert and Richard Nowakowski, A K Peters, Natick, MA, February 2007.

Puzzlers' Tribute: Feast for the Mind, with Tom Rodgers (editors), A K Peters, Natick, MA, December 2001.

Mathematical Go: Chilling Gets the Last Point, with Elwyn Berlekamp, A K Peters, Natick, MA, January 1994. Also in paperback as *Mathematical Go Endgames: Nightmares For the Professional Go Player*, Ishi Press International, San Jose, CA, December 1993. In Japanese as *Igo no Sempo: Yose no Kenkyu*, Translation by Yoshikawa Takeshiro, Toppan, Tokyo, Japan, November 1994.

MICHELE EMMER (University of Rome, Italy)

Michele Emmer is full professor of mathematics at the University of Rome “*La Sapienza*”, *Dipartimento di Matematica*. His area of activity were PDE and minimal surfaces, computer graphics, mathematics and arts, mathematics and culture, films and videos.

Member of the board of the Journal “Leonardo: art, science and technology”, MIT Press Filmmaker, almost all his movies in the series “Art and Math” have been broadcasted by the State Italian television and other television; all the videos are distributed in many countries in the various version (english, french, spanish, italian, japanese). Homage in several Film Festivals: Bellaria, Bergamo, Rome Animation, Torino Museo del cinema, La Villette, Paris, Festival in Pavia, Georges at Centre Pompidou.

He has organized several exhibitions and conferences on the topic of “Art and Mathematics” including the annual conference on “Mathematics and Culture” at the University of Venice; <http://www.mat.uniroma1.it/> ; the exhibitions and conferences on M.C. Escher (1985 and 1998) at the University of Rome; the section on Space at the Biennial of Venice (1986), the travelling exhibition “The Eye of Horus” (Roma, Bologna, Milano, Parma (1989); he was responsible for the Exhibition and congress on “Math and

Art”, Bologna, 2000.

Editor of the series “Mathematics and Culture” by Springer verlag; the series “The Visual Mind” by MIT press; video series “Video math” by Springer verlag.

Last books: “*Visibili armonie arte cinema teatro matematica*”, Bollati Boringhieri, Torino, 2006; “The Visual Mind 2”, MIT Press, 2006; “Mathematics and Culture V”, Springer verlag, 2006; a chapter in “Venise” , Flammarion, Paris, 2006, “matematica e cultura 2007”, Springer, 2007.

He received in 1998 the Galileo award from the Italian Math Association for best popularization of Mathematics. In 2004 the Pitagora award. He is member of the Commission for the popularization of math of the European Math Society.

RICHARD KENNETH GUY (University of Calgary, Canada)

Richard Kenneth Guy (born 1916, Nuneaton, Warwickshire) is a British mathematician, Professor Emeritus in the Department of Mathematics at the University of Calgary.

He is best known for co-authorship (with John Conway and Elwyn Berlekamp) of *Winning Ways for your Mathematical Plays* and authorship of *Unsolved Problems in Number Theory* (ISBN 0-387-94289-0), but he has also published over 100 papers and books covering combinatorial game theory, number theory and graph theory.

He is said to have developed the partially tongue-in-cheek “Strong Law of Small Numbers”, which says there are not enough small integers available for the many tasks assigned to them thus explaining many coincidences and patterns found among numerous cultures.

Additionally, around 1959, Guy discovered a unistable polyhedron having only 19 faces; no such construct with fewer faces has yet been found.

Guy is also a notable figure in the field of chess endgame studies. He composed around 200 studies, and was co-inventor of the Guy-Blandford-Roycroft code for classifying studies. He also served as the endgame study editor for the British Chess Magazine from 1948 to 1951.

Guy is one of the few mathematicians with an Erdos number of 1.

Selected publications:

Richard K. Guy: Aviezri Fraenkel and Combinatorial Games. *Electr. J. Comb.* 8(2): (2001)

Béla Bollobás, Richard K. Guy: Equitable and proportional coloring of trees. *J. Comb. Theory, Ser. B* 34(2): 177-186 (1983)

Richard K. Guy, Gerhard Ringel: Triangular imbedding of $K_n - K_6$. *J. Comb. Theory, Ser. B* 21(2): 140-145 (1976)

RICHARD NOWAKOWSKI (Dalhousie University, Canada)

Education and Employment Information:

B.Sc. (Hons), 1974, University of Calgary; M.Sc. (Mathematics), 1975, University of Calgary (advisor: R.K.Guy); Ph.D. (Mathematics), 1978, University of Calgary (advisor: R.K.Guy). 1998-2003: Chair, Department of Mathematics and Statistics, Dalhousie University. 1992-present: Full Professor, Dalhousie University. 1987-1992: Associate Professor, Dalhousie University. 1983: Awarded tenure, Dalhousie University. 1979-1987: Assistant Professor, Dalhousie University. 1978-1979: Sessional Instructor, University of Calgary.

Research Interests:

Combinatorial games, graph theory, ordered sets.

Books, books edited and chapters in books:

M. Albert, RJN, D. Wolfe, *Lessons in Play*; AK Peters, 2006.

M. Albert, RJN (editors), *More Games of No Chance 3*; to appear, Cambridge University Press, 2007.

RJN (editor), *More Games of No Chance*; Cambridge University Press, 2002.

RJN (editor), *Games of No Chance*; Mathematical Sciences Research Institute Publications 29, Cambridge University Press, 1996.

RJN, "... Welter's game, sylver coinage, dots-and-boxes, ...", *Combinatorial Games, Proceedings of symposia in applied mathematics* 43, 1991; AMS Short Course Lecture Notes, R. K. Guy (editor), pp. 155-182.

THOMAS F. BANCHOFF (Brown University, USA)

Education:

University of Notre Dame, B.A. 1960 (mathematics); University of California, Berkeley, M.A. 1962, Ph.D. 1964 (mathematics)

Teaching Experience:

Benjamin Peirce Instructor, Harvard, 1964-6;
Research Associate, Universiteit van Amsterdam, 1966-7;
Brown University: Asst. Prof. 1967, Assoc. Prof. 1970, Professor 1973;
Walter H. Annenberg Professor of the Year 1998, Brown University;
G. Leonard Baker Visiting Professor of Mathematics, Yale, Fall 1998;
Visiting Professor, University of Notre Dame, Fall 2001;
Visiting Professor, UCLA, Winter 2002;
Visiting Professor, University of Georgia Mathematics Department and College of Education, Spring 2006;
Royce Family Professor in Teaching Excellence, 2005-2008.

Teaching Awards:

Danforth (1960) and Woodrow Wilson (1959) Fellowships;
Senior Teaching Citation, 1976 (chosen by the Brown senior class);
Lester Ford Award 1978 (for outstanding expository writing);
Joseph Priestley Medallion, Dickinson College 1987 (science award);
Bray Award for Teaching Excellence in Sciences, Brown 1993 (faculty);
Mathematical Association of America Northeastern Section Award 1995;
MAA National Award for Distinguished College or University Teaching of Mathematics 1996 (one of three awarded annually);
Doctor of Science, honoris causa, Fairfield University, 1998;
Pew Scholar, Carnegie Foundation Teaching Academy, 1999;
Carnegie Fellow, Carnegie Foundation, 1999-2000;
Doctor of Science, Honoris Causa, Rhode Island College, 2001;
NSF Director's Award for Distinguished Teaching Scholar, 2004.

Publications Directly Related to Teaching:

International Congress of Mathematicians, Helsinki, 1978, invited address in the section on Pedagogy, Computer Animation and the Geometry of Surfaces in 3- and 4-Space (the first such presentation to involve graphics).

Linear Algebra Through Geometry (with John Wermer), Springer-Verlag 1983, revised and expanded second edition 1991.

EDGE-The Educational Differential Geometry Environment, (with Richard Schwartz) 1987 (reworking and expansion of a student final project).

Interactive Geometry and Multivariable Calculus on the Internet, CBMS Issues in Mathematics Education, Volume 14 (2007), 17-31.

Algebraic Thinking and Geometric Thinking, Seventieth Annual NCTM Yearbook (2008).

Beyond the Third Dimension, Scientific American Library 1990, 1996.

Service:

Assoc. Editorships:

Mathematics Magazine 1978-81;

Amer. Math. Monthly 1982-85;

Geometriae Dedicata 1986-95;

Communications in Visual Mathematics, 1996-;

Acting Dean of Student Affairs, Brown, 1970-71;

National Committee: Teaching Assistants and Part-Time Instructors;

Board Memberships:

Society for Values in Higher Education;

CommonCause of RI;

Mathematical Association of America, President, 1999-2000.

3 Program

Wednesday, 29th April

- 8:30 **Welcome and registration**
- 9:00 **Opening Session**
- 9:30 **Rick's Tricky Six Puzzle**
Richard Guy, University of Calgary
- 10:15 **Archimedes's Stomachion**
Henrique Leitão, University of Lisbon
- 10:45 Coffee-Break
- 11:15 **Winning Ways for Mathematically Losing Plays**
Jorge Buescu, University of Lisbon
- 11:45 **Games on Power Networks Dispatch: The World as a Case Study**
José Carlos Quadrado, ISEL
- 12:15 **The Generalisation Squared of Domino**
Cosimo Cardellicchio, University of Bari
- 12:45 Break for lunch
- 14:15 **Math and Films**
Michele Emmer, University of Rome
- 15:00 **Strong Connections Between Chess and Maths**
Alda Carvalho, ISEL
Carlos Santos, ISEC
- 15:30 Coffee-Break
- 16:00 **Piet Hein, a Brilliant Mind**
Natália Bebiano, University of Coimbra
F. J. Craveiro de Carvalho, University of Coimbra
- 16:30 **The Importance of Play**
António Machiavelo, University of Oporto

17:00 **Lozenge Tillings of Polygons**
Ana Paula Cláudio, University of Lisbon
Ilda Perez, University of Lisbon
N. Pinto, University of Lisbon
N. Prehaz, University of Lisbon
J. Soares, University of Lisbon

18:00 Welcome Cocktail

Thursday, 30th April

9:30 **Multiple Pile Take-Away Games**
Aviezri Fraenkel, Weizmann Institute of Science

10:15 **Origami's Geometry**
Liliana Monteiro, EB 2,3 D. António de Ataíde

10:45 Coffee-Break

11:15 **Sangaku**
Antonietta Constantino, ES de Miraflores

11:45 **The Rainbow Game**
Helena Sousa Melo, University of Azores
Ricardo Cunha Teixeira, University of Azores

12:15 **Art Through Functions**
Maria da Graça Marques, University of Algarve
Marília Pires, University of Algarve

12:45 Break for lunch

14:15 Visit to Monsaraz

Friday, 1st May

- 9:30 **Exploring the Fourth Dimension: “Flatland: the Movie”
and the Geometric Art of Salvador Dali**
Thomas Banchoff, Brown University
- 10:15 **Polyhedron Puzzles: Combinatorics and Groups**
Jorge Rezende, University of Lisbon
- 10:45 Coffee-Break
- 11:15 **Mathematical Games for the Blind**
Carlota Dias, University of Lisbon
Jorge Nuno Silva, University of Lisbon
Pedro Palhares, University of Minho
- 11:45 **Card Tricks: Math vs Magic**
Carlos Pereira dos Santos, ISEC
Jorge Nuno Silva, University of Lisbon
- 12:15 **“Zéfiro Explorador” - a Game Designed for Children Focusing Math Skills Training**
Cristiana Vidal Accioly, University of Coimbra
Silvia Barbeiro, University of Coimbra
Aderito Araujo, University of Coimbra
Maria João Santos, University of Coimbra
Michael Diniz, University of Coimbra
- 12:45 Break for lunch
- 14:15 **Gardening and Lawns and Mazes**
Richard Nowakowski, Dalhousie University
- 15:00 **Stamping Patterns**
Arala Chaves, University of Oporto
- 15:30 Coffee-Break
- 16:00 **Medieval Problems (The River-Crossing Puzzle)**
Eurico Nogueira, New University of Lisbon
- 16:30 **A Solitary Pebble Game**
Pedro Duarte, University of Lisbon
- 17:00 **A Very Mathematical Card Trick**
Carlos Santos, ISEC
Jorge Nuno Silva, University of Lisbon
Pedro Duarte, University of Lisbon

Saturday, 2nd May

- 9:30 **In Tribute to Martin Gardner**
David Wolfe, Gustavus Adolphus College
- 10:15 **Discover the Symmetries: an Interdisciplinary Project for the First Grade**
Andreia Hall, University of Aveiro
Carlota Simões, University of Coimbra
Constança Providência, University of Coimbra
- 10:45 Coffee-Break
- 11:15 **Solving Mathematical Puzzles on Saturday Afternoons**
Ana Maria de Almeida, University of Coimbra
- 11:45 **Recreational Mathematics within the classroom: a Portuguese case-study**
Sandra Figo, EB23 Marquês de Pombal
Ana Maria de Almeida, University of Coimbra
- 12:15 **Metromachia - An Ancient Pedagogical Game**
Isabel Catarino, ES Pinhal Novo
- 12:45 Break for lunch
- 14:15 **Mathematical Skills and Mathematical Games**
Dores Ferreira, University of Minho
Jorge Nuno Silva, University of Lisbon
Pedro Palhares, University of Minho
- 15:00 **Descartes's Pedagogical Deck**
Marisa Ferreira, University of Lisbon
Jorge Nuno Silva, University of Lisbon
- 15:30 Closing Session

4 Abstracts

Strong Connections Between Chess and Maths

ALDA CARVALHO, ISEL

CARLOS SANTOS, ISEC

This talk is an overview of the most relevant work that has been made about the connections between chess and mathematics. The examples are chosen to emphasize the chess practice.

Solving Mathematical Puzzles on Saturday Afternoons

ANA MARIA DE ALMEIDA, University of Coimbra

What makes students of all ages want to spend their Saturday afternoons closed inside a 40 years old building solving mathematical quizzes and challenges?

This talk will address a significant experience on recreational mathematics that takes place at the Mathematics Department of the University of Coimbra since 2004, bringing young students from all over the country to spend 2 hours on the company of mathematicians (teachers and undergraduate students). And you know what's also interesting? Many of their parents stay for the play too!

Lozenge Tillings of Polygons

ANA PAULA CLÁUDIO, University of Lisbon

ILDA PEREZ, University of Lisbon

N. PINTO, University of Lisbon

N. PRELHAZ, University of Lisbon

J. SOARES, University of Lisbon

We present some computer games (puzzles) with lozenges. The talk will be focused on some non trivial results of combinatorial geometry concerning tillings of zonogons by lozenges. These results are in general easy to formulate and they were the basis for our algorithms. Natural generalizations lead to some interesting open mathematical and computational questions.

Discover the Symmetries: an Interdisciplinary Project for the First Grade

ANDREIA HALL, University of Aveiro

CARLOTA SIMÕES, University of Coimbra

CONSTANÇA PROVIDÊNCIA, University of Coimbra

The project *Science and Playing - Discover the Symmetries* is taking place in several classes of the 1st cycle of basic education both in Coimbra and Aveiro

and consists in the development and test of educational activities related to the concept of symmetry with groups of young children. Concepts such as translation, rotation and reflection in the plane are introduced with the help of friezes, mirrors or by folding or cutting paper. Symmetries in time are also studied, by the transformation of basic melodies. The concept of pavement appears naturally with the creation of mosaics by means of a regular figure, like the triangle or a hexagon. Techniques for building mosaics from irregular figures are also given. Chirality, absence of symmetry, points of stability and unstable equilibrium points are introduced by the construction, in the classroom, of learning toys. The concept of axis of symmetry and degree of symmetry in three-dimensional figures is presented together with the study of minerals. In this communication we reveal some of the experiences that have been tested among children during this school year under this project.

Sangaku

ANTONIETA CONSTANTINO, ES de Miraflores

Abstract: *Sangaku* are Japanese geometrical puzzles painted on wooden tablets created during the Edo period (1603-1867), when Japan was isolated from the rest of the world. The *Sangaku* hung in the precincts of shrines as offerings to the gods.



The Importance of Play

ANTÓNIO MACHIAVELO, University of Oporto

In this talk we will briefly digress on the importance of play, from an evolutionary and ethological point of view. We will then talk on the role that mathematical recreations had, and could have, for educational purposes, as well as for Mathematics itself.

Stamping Patterns

ARALA CHAVES, University of Oporto

There are several proofs of the well known facts that there are exactly 17 doubly periodic patterns and 7 kinds of friezes in the plane modulo symmetry.

There are several proofs of the well known facts that there are exactly 17 doubly periodic patterns and 7 kinds of friezes in the plane modulo symmetry.

An elegant geometric approach to these results uses the theory of *orbifolds* (W. Thurston). In this talk we will survey the DVD produced by Atractor, which has as main goal to treat that approach to symmetry in a non technical way. Given a pattern or frieze its symmetries are detected and a stamp is produced. To each of these stamps, with its own structure (boundaries, conical points and corners) an invariant is associated - the Euler number. The good stamps for the plane being those with Euler number 0. We will show how, by means of a “game”, it is possible to find all the good stamps; then, starting with each stamp, it is possible to generate new patterns and friezes with chosen motifs and watch how its symmetries behave under affine perturbations.

Multiple Pile Take-Away Games

AVIEZRI FRAENKEL, Weizmann Institute of Science

The class of impartial take-away games appears to be partitioned into two disjoint subclasses: those that are easy to generalize to more than one or two piles, and those for which this seems to be very hard. A well-known representative of the former is Nim, and of the latter, Wythoff’s game. Some progress in generalizing Wythoff to multiple piles was recently made. Two 3-pile games that are *extensions* rather than generalizations of Wythoff were also given recently. Here we study an extension of Wythoff to four piles. The efforts in defining a “right” extension, and particularly in proving the validity of the winning strategy, are considerably greater than those for three-pile extensions. We present four winning strategies, recursive, algebraic, arithmetic and morpic. The recursive is the easiest to describe, but seems to be hardest computation- ally. The algebraic depends on iterations of the floor function, the arithmetic on the Fibonacci numeration system and the morpic on a morphism. The latter three are polynomial time winning strategies.

Card Tricks: Math vs Magic

CARLOS PEREIRA DOS SANTOS, ISEC

JORGE NUNO SILVA, University of Lisbon

There are many kinds of card tricks. Magicians can execute some of them

very quickly, however, often they must cheat. There is a different class of card tricks made by mathematicians. Those self-working card tricks can be very beautiful. We will present some math-tricks. Our assistants will be Cantor, Erdos, Leibniz, Dirichlet,

A Very Mathematical Card Trick

CARLOS PEREIRA DOS SANTOS, ISEC

JORGE NUNO SILVA, University of Lisbon

PEDRO DUARTE, University of Lisbon

An Andy Liu's paper, *Two Applications of Hamming Code* (Math. Mag.), studies a very interesting self-working card trick involving Hamming Codes. This trick has a very interesting geometric approach, and Combinatorial Game Theory provides the best way to execute it.

Mathematical Games for the Blind

CARLOTA DIAS, University of Lisbon

JORGE NUNO SILVA, University of Lisbon

PEDRO PALHARES, University of Minho

Playing games develops concentration, the practice of obeying rules, the establishment of plans and the analysis of reasoning lines behind a decision. This process leads the player to consider an object not as a physical thing but as something that is abstract and has certain rules (Silva and Neto, 2006). Mathematical games research related to education carried out in Portugal, have shown that there is a relationship between the strength of play, particularly in the chess game, and mathematical skills related to solving problem, when people follow patterns; (Ferreira and Palhares, 2008) and there are also indications that other mathematical games have similar relationship (Ferreira, Palhares and Silva 2008). However, in Portugal there are no records of studies in the area of mathematical games directed to children and / or young people with poor vision or blindness.

The project "Mathematical Games for the Blind" intends to initiate such study. Its aim is the selection and adaptation of mathematical games for children with poor vision or blindness, so that they can participate in the National Championship of Mathematical Games. The first step of this study was the creation of prototypes for some games, according to the Educational Research Design Method that was adopted. After a detailed study in order to implement this new process, a new "logout" has been created. The second step pretends to be a qualitative study of the different skills that can be stimulated in children with poor vision or blindness, when they practice these games.

A summary of this project, its aims, the process and the prototypes that have been used as well as all the feedback we got, will be the contents of this communication.

The Generalisation Squared of Domino

COSIMO CARDELLICCHIO, University of Bari

Polyominoes are a milestone in recreational mathematics since the first work on this topic by Solomon Golomb. Golomb proposed his nomenclature starting from a nice word-play in which a domino is a two square form (from the latin duo = two), and thus every poly-square assembly should be a “polyomino”.

The Golomb sentence that polyominoes are a generalisation of domino is able to generate further generalisations, in a certain sense generalisations “squared” of the original concept. A brief survey of these tiles and their application in commercial games will be presented.

“Zéfiro Explorador” - a Game Designed for Children Focusing Math Skills Training

CRISTIANA VIDAL ACCIOLY, University of Coimbra

SILVIA BARBEIRO, University of Coimbra

ADÉRITO ARAUJO, University of Coimbra

MARIA JOÃO SANTOS, University of Coimbra

MICHAEL DINIZ, University of Coimbra

The objective of this project was to construct an instrument to work mathematical contents with children, specially 5th grade students. For this purpose, the game was created with the following concerns: to be adjusted to the 5th grade mathematical curriculum plan; to cover different concepts of mathematics; set math problems in a fantasy kids’ world to be fun and attractive; to be able to be played at home or at the classroom; to provide interaction between the groups of players; to rise up discussions about how to solve the problems.

The game, named Zéfiro the explorer, consists of a board with a map appearance which represents six math worlds. The themes of these worlds are geometry, astronomy, arithmetic, fractions and weights and measures. The last world puts together these areas of knowledge. Children are invited to join the character and become explorers. The trail, from the beginning through the end, is full of enigmas and sends “back” and “forward” messages to increase the interest of the game. A dice dictates the fortune of the players but in their journey they have to accomplish the challenges proposed by enigma cards or learn from informative cards.

“Zéfiro Explorador” was a result of a collaboration between the Departamento de Matemática da Universidade de Coimbra and the Hospital Pediátrico de Coimbra in the scope of the more wide project called “Aprender a Brincar”, which aimed to bring science in a playfully way to the children of the hospital’s nursery. The game was tested in the hospital, primary schools, meetings and in a “Ciência Viva” exhibition.

The knowledge generated by this study will make possible to identify its educational potential. Preliminary findings indicate that the game stimulates the learning in group, potentiate the interest of children solving math problems and had a good acceptance by teachers and pedagogues to construct alternative ways of learning.

In Tribute to Martin Gardner

DAVID WOLFE, Gustavus Adolphus College

The field of recreational mathematics includes games, logic, and puzzles, magic squares and fractals. Much of the area is approachable to the non-specialist, as is evidenced by Martin Gardner’s column in Scientific American spanning 3 decades.

Once every few years, a group of mathematicians, magicians and puzzle designers assemble in honor of Martin Gardner, many of whom were strongly influenced by Martin’s writings in all these areas. All participants bring contributions to share with the group. At this talk, I will tell some stories from the event, and I will present my past contributions which include logic applied to Scrabble puzzles, curious properties of pandigital numbers (numbers made up of the digits 0-9 anagrammed), and some paradoxes about money.

This talk will be approachable to a very broad audience, and only high school algebra is required.

Mathematical Skills and Mathematical Games

DORES FERREIRA, University of Minho

JORGE NUNO SILVA, University of Lisbon

PEDRO PALHARES, University of Minho

Playing games is one of the most appreciated activity in the occupation of adults and especially children leisure time. This makes playing games one of the motivating teaching strategies. Besides, the use of mathematical games has been cited as a facilitator of development of mathematical skills (DEB, 2001), including the logical (Smole, Diniz and Cândido, 2007).

The ability to identify patterns is related to diverse areas of mathematics. Some even consider mathematics as the science of patterns (Devlin, 1997; Steen, 1990).

Studies have shown that the ability to identify patterns is related to the game of chess (Ferreira and Palhares, 2007, 2008), and the game of traffic lights (Ferreira, Palhares and Silva, 2008), in elementary school students. In this presentation we intend to disclose more recent results of an ongoing study with children from the 3rd and 4th years of elementary education, where other games were selected to verify their relation to the identification of patterns.

Medieval Problems (The River-Crossing Puzzle)

EURICO NOGUEIRA, New University of Lisbon

In this talk I will give an historical account of the river-crossing problems since its beginnings with Alcuin of York, give its solutions and relate them with other classical problems such as “The four elopements” or “Missionaries and Cannibals”.

The Rainbow Game

HELENA SOUSA MELO, University of Azores

RICARDO CUNHA TEIXEIRA, University of Azores

A new game is presented. The Rainbow Game is a two-player abstract game inspired by the Traffic Lights Game, created by Alan Parr, but using the seven colors of the rainbow and with some specifications.

Players place counters on a hexagonal grid, either laying red counters on empty spaces or upgrading red to orange, orange to yellow, yellow to green, and so on, following the seven colors of the rainbow.

Rainbow is a cunning game, playable at several levels of complexity. One possible goal is to complete a specific type of connection with the color’s pieces, e.g. to join the seven colors in a connected path satisfying some properties. Some winning strategies are explored.

Archimedes’s Stomachion

HENRIQUE LEITÃO, University of Lisbon

Archimedes’s “Stomachion” is one of the lesser works of the famous mathematician from Ancient Greece. The convoluted history of its textual transmission combined with the difficulty in interpreting the fragments of extant text has obscured the true meaning of the work; usually it has been in-

terpreted as some sort of game. However, paleographic findings and new interpretations in recent years seem to have shed a new light on this strange and difficult text. In this presentation I will summarize the history of this text commenting on its possible interest for the history of board games.

Metromachia - an ancient pedagogical game

ISABEL CATARINO, ES Pinhal Novo

Metromachia is a game that unfolds in an imaginary war scenario, deeply influenced by medieval ideas, even though it was a time of liberation from obscure concepts. Two armies tried to defend their castles and conquer their enemy's. Metromachia can be apprehended as a war game included in the family of the abstract games.

Metromachia was a game where all the elements of the two armies fighting each other were represented by plane figure and geometric solids. The hierarchy was characterized by the attributes of each piece.

As a game where two geometrical armies face each other, all moves, blockages and captures are grounded on mathematical concepts and knowledge, namely arithmetic and geometry.

The *μετρομαχια*, *sive Ludus Geometricus*, it's a game created in 1578 by William Fulke, professor in Cambridge.

Winning Ways for Mathematically Losing Plays

JORGE BUESCU, University of Lisbon

Ten years ago the Spanish physicist Juan Parrondo introduced a strikingly counter-intuitive paradox involving games of chance, now known as Parrondo's paradox. We consider two fair games A and B, both of which can be turned into games with a losing expectation by changing a parameter. Even though A and B are now losing games, when they are played in alternate (even random) order a winning expectation for the outcome results. We also describe the physical origin of this paradox, as well as some of its (possible) applications.

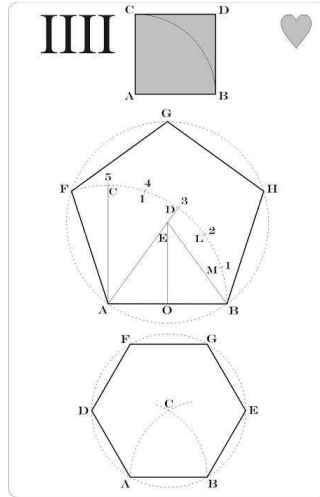
Descartes's Pedagogical Deck

JORGE NUNO SILVA, University of Lisbon

MARISA FERREIRA, University of Lisbon

In 1697 James Moxon, son of Joseph Moxon, an important publisher and author who had been hydrographer of Charles II and member of RAS, published *The Use of Geometrical Playing Cards*, a translation of a French manuscript

by René Descartes. This is a book focused on Geometry, with a beautiful deck of 52 cards illustrating the mathematical propositions. But, is it really due to Descartes?



Polyhedron puzzles: Combinatorics and Groups

JORGE REZENDE, University of Lisbon

In this talk we shall discuss the construction of the polyhedron puzzles and, at a very elementary level, the mathematical concepts involved and the possible scientific developments. We shall treat the following topics:

1. Construction of the puzzles using combinatorics.
2. Polyhedron puzzles, symmetric groups and isometry groups.
3. Construction of the puzzles using group theory.
4. Existence of solutions and the problem of counting them.
5. Generalizations.

Games on Power Networks Dispatch: The World as a Case Study

JOSÉ CARLOS QUADRADO, ISEL

The object of games on Power Networks Dispatch is to supply the most consumers with power. Usually, players mark pre-existing routes between cities for connections, and then bid against each other to purchase the power

plants that they use to power their consumers.

However, as power plants are purchased, newer and more efficient plants become available, so by merely purchasing a certain technology the players are potentially allowing others access to superior equipment.

Additionally, players must acquire the raw materials (e.g. coal, oil, garbage and uranium) needed to power those plants, making it a constant struggle to upgrade the plants for a maximum efficiency while still retaining enough wealth to quickly expand their network to get the cheapest routes. New vectors come into action when these games include “renewable” wind / solar plants which require no fuel, or even when they include the future nuclear fusion.

Besides the dramatic behavior that these sorts of games allow by emulating the energy markets, the process can be even more singular when they incorporate the case studies of different regions of the globe. These change the game by, not only changing the characteristics of the geography, but also by changing the rules for power plant availability, resource availability, tying plant ownership and/or resource availability and/or price to geographic regions on the map. When considering some very peculiar regions like Italy, France, USA, Germany, Benelux, Central Europe, Korea and China these games become much more than just games and convert the world into a case study in the field of energy production, transport and usage.

Origami's Geometry

LILIANA MONTEIRO, EB 2,3 D. António de Ataíde

Origami is the famous Japanese art of folding paper. In this presentation you will find out that there are essentially only seven possibilities for one single Origami fold. These form the Huzita-Hatory Axioms for Origami, and they will be shown to you by the interactive geometry program Cinderella. You will also see that this geometry goes beyond Euclidean Geometry, solving the classic problems of trisecting an angle and doubling a cube.

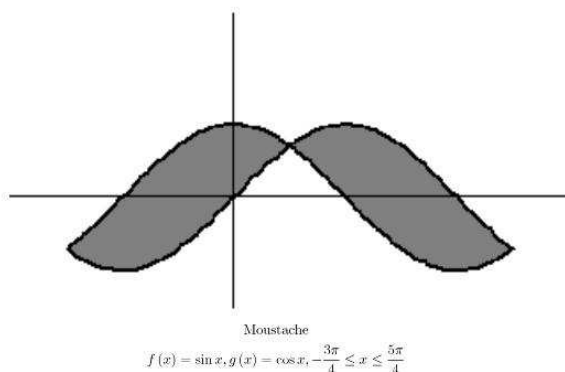
Art Through Functions

MARIA DA GRAÇA MARQUES, University of Algarve

MARÍLIA PIRES, University of Algarve

In the last years some mathematical software applications combining geometry, algebra and calculus have been developed. These applications can handle both the graphic and analytical expression of functions. The aim of this work is to show how these software applications can be used to produce some artistic works using graphs of functions and how this can be an efficient method

for teaching functions, namely for making students understand, in a recreational way, how the change of parameters in the analytical expression of a function affects the shape of its graph and also make them discover many other facts concerning functions and their graphs. To illustrate roughly the idea, we present below a simple example of a moustache built from two well-known functions.



Math and Films

MICHELE EMMER, University of Rome

In the last years there has been an increasing interest in cinema on the stories related with mathematicians, both of real mathematicians and of fiction characters. In my presentation I will compare the role of the mathematicians, both real and fictitious, in the films of the last decade characterizing the role that mathematicians have in films. In particular their role in detective and crime stories, in biographies of real mathematicians, in fiction and even in comedies. I will consider films from Italy, France, England, Spain, Germany, USA. I will also show some of my movies, like “Flatland”, “M. C. Escher”, “Soap Bubbles”.

Piet Hein, a Brilliant Mind

NATÁLIA BEBIANO, University of Coimbra

F. J. CRAVEIRO DE CARVALHO, University of Coimbra

Piet Hein was an inventive and impressive personality of the twentieth century. In this communication we present two distinct facets of his work, including some of his famous grooks and games.

A Solitary Pebble Game

PEDRO DUARTE, University of Lisbon

We describe the solution to a Pebble Solitaire Game of M. Kontsevich through elementary linear algebra techniques.

Rick's Tricky Six Puzzle

RICHARD GUY, University of Calgary

Roughly speaking, Rick Wilson's theorem says that in a sliding block puzzle, you can reach all possible positions, or exactly half of them; the classical example of the latter case is the Fifteen Puzzle. However, there is an exception: Rick's Tricky Six Puzzle, in which you can reach only $5!$ out of the $6!$ possible positions. It's not much of a puzzle, but it has amazing connexions with far more things than I'll have time to tell you about: shuffling a deck of cards, the projective plane of order 4, the Hoffman-Singleton graph, the Steiner system $S(5, 6, 12)$, binary and ternary error-correcting codes, . . . : for more detail see the current, April 2009, issue of *Mathematics Magazine* for an article with Alex Fink.

Gardening and Lawns and Mazes

RICHARD NOWAKOWSKI, Dalhousie University

The English have been noted for (are obsessed with) keeping a tidy garden ready for games. I'll look at the (mathematical versions of) games: where a good lawn is needed (lawn bowling); where a perfect lawn is required (cricket!); and where hedges need to be pruned (Hackenbush); and when they are turned into Mazes.

Recreational Mathematics within the Classroom: a Portuguese Case-Study

SANDRA FIGO, EB23 Marquês de Pombal

ANA MARIA DE ALMEIDA, University of Coimbra

This talk will begin by presenting a three year long experience with a Portuguese class that one of the authors has been teaching during the important years that precede secondary school (7th to 9th grades of Portuguese Basic School, which means 12 to 14/15 years old students). We discuss their progress at Mathematics, basically at the level of their attitude towards the discipline, where they were confronted with specially designed exercises, most of the times with a recreational flavor, as a differentiating factor from having only textbook exercises to practice the learning subjects. We will show, not only some of the proposed exercises, but also some of the students resolutions. The presentation will continue showing this school year experience with two different 7th Portuguese Basic grade classes and will proceed

by presenting some of the comments made either by the class that was been experiencing this kind of learning tool and class environment during the last three years, either by the two new classes (which possess quite different attitudes towards learning in general). We will end by a brief reflection upon this work, which we believe that should continue, and show the gratifying results that, as teachers, were obtained.

*Exploring the Fourth Dimension: "Flatland: the Movie"
and the Geometric Art of Salvador Dali*

THOMAS BANCHOFF, Brown University

In the new movie version of "Flatland", the key geometric figure is a cube that rotates about its center in three-space producing a number of central slices in the plane including a square, a rectangle, a rhombus, and a hexagon. What are the analogous central hyperplane slices in our three-dimensional space of a hypercube rotating about a point in four-dimensional space? In Salvador Dali's "Corpus Hypercubus", the hypercube appears in a fold-out form, inspired by the combinatorial philosophy of Ramon Lull. How can we explore such representations of the hypercube using interactive computer graphics, and produce materials that can be used by teachers and students at all levels?

5 Mailing List

Alda Carvalho	ISEL, Portugal	acarvalho@dem.isel.ipl.pt
Aleksandar Kostadinov	University of Évora, Portugal	alexandarkostadinov@yahoo.com
Alexandra Bento	EB 2,3 de Pegões, Portugal	a.cunha.bento@gmail.com
Alice Faro e Santos	ES Laura Ayres, Portugal	alice_faro@yahoo.com
Ana Correia	ES Laura Ayres, Portugal	nani.correia@megamail.pt
Ana Maria de Almeida	University of Coimbra, Portugal	amca@mat.uc.pt
Ana Paula Cláudio	University of Lisbon, Portugal	apc@di.fc.ul.pt
Ana Paula Silva	ES da Moita, Portugal	anapfsilva@netcabo.pt
Ana Rita Castanheira	Portugal	anacastanheira@gmail.com
Andreia Frazão	University of Coimbra, Portugal	andreiafrazao@iol.pt
Antonietta Constantino	ES Miraflares, Portugal	maria.a.c.c@gmail.com
António Machiavelo	University of Oporto, Portugal	ajmachia@fc.up.pt
António Manilhas	EBI André de Resende, Portugal	antonio.manilhas@netvisao.pt
Arala Chaves	University of Oporto, Portugal	machaves@fc.up.pt
Aviezri Fraenkel	Weizmann Institute of Science, Israel	aviezri.fraenkel@weizmann.ac.il
Carla Anjos	EB 2,3 Amadeu Gaudêncio da Nazaré, Portugal	dosanjscarla@gmail.com
Carla Costa	Escola 2,3 Dr. Armanado Lizardo, Portugal	cmfc77@gmail.com
Carlos Santos	ISEC, Portugal	cps.carlos@gmail.com
Carlota Dias	EB3S de Matias Aires, Portugal	carlotadia@gmail.com
Carlota Simões	University of Coimbra, Portugal	carlota@mat.uc.pt
Carmen Barreiras	Inovinter, Portugal	carmenbarreiras@sapo.pt
Catarina Pinheiro	University of Évora, Portugal	txica.n_n@hotmail.com
Cosimo Cardellicchio	University of Bari, Italy	ccardel@alice.it
Cristiana Accioly	University of Coimbra, Portugal	cristiana.accioly@gmail.com
Daniela Gomes	ES Frei Rosa Viterbo, Portugal	danielagomes@escolarosaviterbo.pt
David Wolfe	Gustavus Adolphus College, Canada	davidgameswolfe@gmail.com
Evanina Martins	EB Sebastião da Gama, Portugal	evanina.martins@gmail.com
Eurico Nogueira	New University of Lisbon, Portugal	jen@fet.unl.pt
Filipe Papança	University of Évora, Portugal	filipe.papanca@gmail.com
Guida Saruga	EBI André de Resende, Portugal	guidal4@gmail.com
Helena Melo	University of Azores, Portugal	hmelo@uac.pt
Henrique Leitão	University of Lisbon, Portugal	leitao.henrique@gmail.com
Ilda Perez	University of Lisbon, Portugal	isilva@cii.fc.ul.pt
Isabel Castanheira	EB 2,3 Ramalho Ortigão, Portugal	isabel_castanheira@msn.com
Isabel Catarino	ES de Pinhal Novo, Portugal	isabel.catarino@mail.telepac.pt
Joana Grade	University of Évora, Portugal	bacana_@msn.com
João Machado	University of Évora, Portugal	jfnsmachado@gmail.com
João Mulas	ES de Montemor-o-Novo, Portugal	joaomulas@gmail.com
João Pedro Neto	University of Lisbon, Portugal	jpn@di.fc.ul.pt
João Soares	University of Lisbon, Portugal	joaomcsoares@gmail.com
Jorge Buescu	University of Lisbon, Portugal	jbuescu@gmail.com
Jorge Nuno Silva	University of Lisbon, Portugal	jnsilva@cal.berkeley.edu
Jorge Rezende	University of Lisbon, Portugal	jorgerezend@gmail.com
José Carlos Quadrado	ISEL, Portugal	jqquadrado@isel.ipl.pt
Liliana Monteiro	EB 2,3 D. António de Ataíde, Portugal	lilianamonteiro@net.sapo.pt
Luis Guiomar	EB 2,3 de Cacia, Portugal	lj_guiomar@sapo.pt
Luis Malheiro	ES de Odivelas, Portugal	malheirol@gmail.com
Luisa Carvalho	University of Évora, Portugal	carvalh.luisa@gmail.com
Maria da Graça Marques	University of Algarve, Portugal	gmarques@ualg.pt
Maria das Dores Ferreira	University of Minho, Portugal	doresferreira@iec.uminho.pt
Maria de Fátima Morgado	EBI André de Resende, Portugal	romaofatima@sapo.pt
Maria Gabriela Branquinho	EBI André de Resende, Portugal	mariagbranquinho@gmail.com
Maria Helena Rosmaninho	EBI André de Resende, Portugal	hfrosmaninho@gmail.com
Maria João Santos	Portugal	mjosantos@gmail.com
Marília Pires	University of Algarve, Portugal	mpires@ualg.pt
Mário Santos	PENGEST, Portugal	qas@pengest.pt
Marta Melgão	University of Évora, Portugal	martamelgao@gmail.com
Michael Diniz	University of Coimbra, Portugal	michael.diniz@yahoo.com.br
Michele Emmer	University of Rome, Italy	emmer@mat.uniroma1.it
Nátalia Bebiano	University of Coimbra, Portugal	bebiano@mat.uc.pt
Natasha Borrega	University of Évora, Portugal	pixis934@gmail.com
Nelson Pinto	University of Lisbon, Portugal	nelson nrap@hotmail.com
Nuno Prelhaz	University of Lisbon, Portugal	ninjasistem@hotmail.com
Paulo Infante	University of Évora, Portugal	pinfante@uevora.pt
Pedro Duarte	University of Lisbon, Portugal	pduarte@ptmat.fc.ul.pt
Pedro Palhares	University of Minho, Portugal	palhares@iec.uminho.pt
Ricardo Teixeira	University of Azores, Portugal	rteixeira@uac.pt
Richard Guy	University of Calgary, Canada	rkrg@cpsc.ucalgary.ca
Richard Nowakowski	Dalhousie university, Canada	rjn@mathstat.dal.ca
Rui Dias	University of Évora, Portugal	RuiDias666@gmail.com
Sandra Lagarto	University of Évora, Portugal	sdlagarto@gmail.com
Sandra Figo	EB 2,3 Marqués de Pombal, Portugal	sandrafigo@clix.pt
Sandra Vinagre	University of Évora, Portugal	smv@uevora.pt
Sara Silva	University of Évora, Portugal	astronauta.sara@gmail.com
Susana Fernandes	University of Évora, Portugal	sudf@portugalmail.pt
Susana Santos	EP de Desenvolvimento Rural de Serpa, Portugal	su.santos121@gmail.com
Thomas Banchoff	Brown University, USA	Thomas_Banchoff@brown.edu
Verónica Filipe	Portugal	veronica.filipe@clix.pt